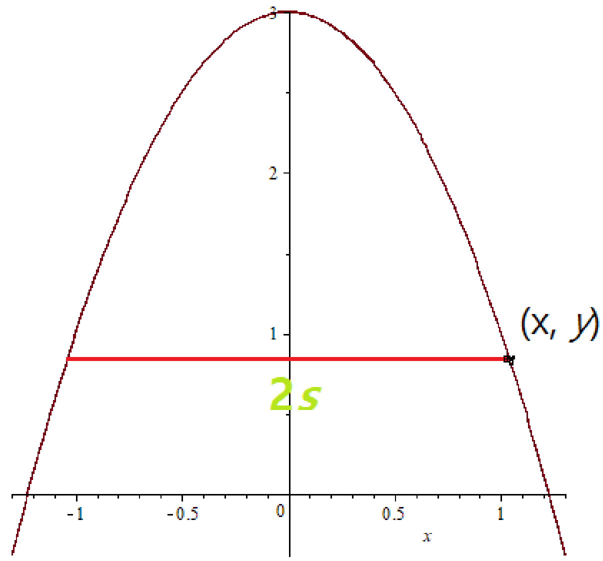
Q: Find the volume of the solid whose base is inclosed by the parabola $y = 3 - 2x^2$ and whose cross-sections are squares perpendicular to the y-axis.

A: We divide the solid into an infinite number of infinitessimally thin square slices perpendicular to the y-axis. Each slice has volume

$$\mathrm{d}V = S^2 \mathrm{d}y$$

where S is the side length.



We can see from the figure that for a given point (x, y) on the parabola, the corresponding side length will be $S = 2x$. Solving $y = 3 - 2x^2$ for x gives

$$x = \sqrt{\frac{3}{2} - \frac{y}{2}}$$

so that

$$S = 2\sqrt{\frac{3}{2} - \frac{y}{2}}$$ and

$$\mathrm{d}V = 4\left(\frac{3}{2} - \frac{y}{2}\right)\mathrm{d}y = (6 - 2y)\mathrm{d}y$$

Integrating these volume elements from the lowest to highest y-values gives

$$V = \int_0^3 (6 - 2y)\mathrm{d}y = \left[6y - y^2\right]_0^3 = \boxed{9}$$